An Efficient Re-Analysis Methodology for Vibration of Large-Scale Structures

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ABSTRACT

Finite element analysis (FEA) is a well-established methodology in structural dynamics. However, optimization and/or probabilistic studies can be prohibitively expensive because they require repeated FEA of large models. Various re-analysis methods have been proposed in order to calculate efficiently the dynamic response of a structure after a baseline design has been modified, without recalculating the new response. The parametric reduced-order modeling (PROM) and the combined approximation (CA) methods are two re-analysis methods, which can handle large model parameter changes in a relatively efficient manner. Although both methods are promising by themselves, they can not handle large FE models with large numbers of degrees of freedom (DOF) (e.g. 100,000) and design parameters (e.g. 50), which are common in practice. In this paper, a new re-analysis method is proposed where the original CA method is modified to further improve its efficiency, especially for problems where a large number of modes is required. The modified CA (MCA) method is then integrated with the PROM approach, in order to formulate a new efficient re-analysis method that is suitable for large FE models with many design parameters that vary in a wide range. A simple frame structure is used to explain all steps of the proposed method. Also, a vibro-acoustic analysis of a realistic vehicle FE model is presented to demonstrate the efficiency and accuracy of the new method. A design optimization study is also performed to highlight the accuracy and efficiency of the proposed re-analysis method.

1. INTRODUCTION

Finite element analysis (FEA) is a well-established numerical simulation method for structural dynamics. It serves as the main computational tool for NVH analysis in the low-frequency range. As a result of developments in numerical methods, as well as advances in computer software and hardware, FEA can now handle much more complex models far more efficiently than even a few years ago. However, the demand for computational capabilities increases in step with, or even beyond the pace of these improvements. For example, automotive companies are constructing more detailed models with millions of degrees of freedom (DOF) to study vibro-acoustic problems in higher frequency ranges. Although these tasks can be performed with FEA, the computational costs are enormous, even for high-end workstations with the most advanced software.

There are also other factors that increase the computational costs. When design changes are involved, the FEA analysis must be repeated many times in order to obtain the optimum design. Furthermore in probabilistic analysis where parameter uncertainties are present, the FEA analysis must be repeated for a large number of sample points. In such cases, the computational cost is even higher, if not prohibitive.

Re-analysis methods are intended to analyze efficiently structures that are modified due to various changes. The objective is to evaluate the structural response for such changes without solving the complete set of modified analysis equations. Several reviews have been published on re-analysis methods [1-3]. These methods are usually based on local and global approximations. Local approximations are very efficient but they are effective only for small structural changes. Global approximations are preferable for large changes, but they are usually computationally

expensive especially for cases with many design parameters. The well-known Rayleigh-Ritz re-analysis procedure [4, 5] belongs to the category of local approximation methods. The mode shapes of a nominal design can be used to form a Ritz basis for predicting the response in a small parametric zone around the nominal design point. However, it is incapable of capturing relatively large design changes.

A parametric reduced-order modeling (PROM) method, developed by Balmes [6, 7], expands on the Rayleigh-Ritz method, by using the mode shapes from a few selected design points to predict the response throughout the design space. The PROM method still belongs to the category of local approximation methods, but it can handle relatively larger parameter changes because it uses multiple design points. An improved component-based PROM method has been introduced by Zhang et. al. [8, 9] for design changes at the component level. The method was implemented in NASTRAN DMAP (Direct Matrix Abstraction Program), and its efficiency and accuracy were demonstrated using a medium size, finite-element vehicle model.

The PROM method using a ‘parametric’ approach (Section 2.2), has been successfully applied to design optimization and probabilistic analysis of vehicle structures. However, the ‘parametric’ approach is only applicable to problems where the mass and stiffness matrices can be approximated by a polynomial function of the design parameters and their powers. A new ‘parametric’ approach using Kriging interpolation [10] has been recently proposed [11]. It improves efficiency by interpolating the reduced system matrices without needing to assume a polynomial relationship of the system matrices with respect to the design parameters as in [6, 7].

The Combined Approximations (CA) method [12-14] combines the strengths of both local and global approximations, being accurate even for large design changes. The CA method is a

combination of binomial series (local) approximations (also called Neumann expansion approximations) and reduced basis (global) approximations. Originally, the CA method was developed for linear static re-analysis. It has also been extended to eigen-problem re-analysis [15-19]. Accurate results and significant computational savings have been reported. A comparative study [20] has shown that the method is suitable for large design changes.

All reported studies on the CA method [12-19], have used relatively simple frame or truss systems for static or dynamics analysis with a relatively small number of DOF and/or modes. For these problems, the computational efficiency was improved by a factor of 5 to 10. Such an improvement is beneficial for a single design change evaluation or even for gradient-based design optimization, where only a limited number of re-analyses (e.g. less than 50) is performed. However, the computational efficiency of the CA method may not be adequate in simulation-based (e.g. Monte-Carlo) probabilistic dynamic analysis of large finite-element models, where re-analysis must be performed hundreds or thousands of times in order to estimate accurately the reliability of a design. The main reason that the CA method is still expensive for such problems is the considerable cost for calculating the basis vectors used to approximate the response.

A large number of modes must be calculated and used in a dynamic analysis of a large finite-element model with a high modal density, even if a reduced-order modeling approach is used (Section 2.1). In such a case, the implicit assumption of the CA method that the cost of solving a linear system is dominated by the cost of matrix decomposition is no longer valid (see section 3.1). As a result, the computational savings from using the CA method may not be substantial. For this reason, a modified combined approximation (MCA) method is proposed in this paper and is integrated with the PROM method for improving accuracy and computational efficiency.

The MCA method approximates efficiently the modes of a number of designs. These modes are subsequently used by the PROM method in an optimization process. The computational savings can be substantial for problems with a large number of design parameters. Examples demonstrate the benefits of the proposed re-analysis method.

The paper is organized as follows. The existing PROM and CA methods are reviewed in Section 2 after a brief introduction to reduced order modeling for dynamic systems. Section 3 describes the proposed MCA method and its integration with gradient-based optimization and the PROM method. Section 4 demonstrates the advantages of the proposed MCA method and its integration with optimization and the PROM method, using a simple frame structure example and a complex vehicle model. Finally, conclusions are drawn in Section 5.

2. REVIEW OF EXISTING METHODS

2.1 Reduced Order Modeling for Dynamic Response

For an undamped structure with stiffness and mass matrices $\mathbf{K}$, and $\mathbf{M}$ respectively, under the excitation force vector $\mathbf{F}$, the equations of motion (EOM) for frequency response are

$$[\mathbf{K} - \omega^2 \mathbf{M}]\mathbf{d} = \mathbf{F} \tag{1}$$

where the displacement $\mathbf{d}$ is calculated at the forcing frequency ω. If the response is required at multiple frequencies, the repeated direct solution of Eq. (1) is computationally very expensive and therefore, impractical for large scale finite-element models.

The reduced order model (ROM) of this Section is a subspace projection method. Instead of solving the original response equations, it is assumed that the solution can be approximated in a

subspace spanned by the dominant mode shapes. A modal response approach can be used to calculate the response more efficiently. A set of eigen-frequencies ω_i and corresponding eigenvectors (mode shapes) $\boldsymbol{\varphi}_i$ are first obtained. Then, the displacement **d** is approximated in the reduced space formed by the first n dominant modes as

$$\mathbf{d} = \hat{\boldsymbol{\Phi}}\mathbf{U}, \tag{2}$$

where $\hat{\boldsymbol{\Phi}} = \begin{bmatrix}\boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \cdots & \boldsymbol{\varphi}_n\end{bmatrix}$ is the modal basis and U is the vector of principal coordinates or modal degrees of freedom (DOF). Thus, the EOM of Eq. (1) can be transformed from the original physical to the modal degrees of freedom as

$$\left[\hat{\boldsymbol{\Phi}}^T\mathbf{K}\hat{\boldsymbol{\Phi}} - \omega^2\hat{\boldsymbol{\Phi}}^T\mathbf{M}\hat{\boldsymbol{\Phi}}\right]\mathbf{U} = \hat{\boldsymbol{\Phi}}^T\mathbf{F}. \tag{3}$$

Solving for the modal response **U** and projecting it back to the physical coordinates, the response **d** can be recovered. If $\omega_{\max}$ is the maximum excitation frequency, it is common practice to retain the mode shapes in the frequency range of $0 - 2\omega_{\max}$. The system modes in the high frequency range can be safely truncated with minimal loss of accuracy.

Due to the modal truncation, the size of the ROM is drastically reduced, compared to the original model. However, the size increases with the maximum excitation frequency. An added benefit of the ROM is that Eqs. (3) are decoupled because of the orthogonality of the mode shapes. Therefore these equations can be solved separately, reducing further the overall computational effort.

2.2 Parametric Reduced-Order Modeling Method

The parametric reduced-order modeling (PROM) method is a subspace approximation method developed to improve the efficiency of re-analysis. It assumes that the dynamic response of a new design, which is obtained by relatively large parameter changes from the baseline design, can be approximated in the subspace spanned by the mode shapes of some representative designs.

The quality of the approximation depends on the selection of the representative designs. Balmes et al. [6, 7] suggested a procedure for obtaining such designs. Consider a design defined in terms of m design parameters $\mathbf{p}$ such as plate thicknesses, spot weld diameters, spot weld pitch of a car body, etc. The ranges in which these parameters and their baseline values can vary are estimated by the designer. We consider $(m+1)$ designs where the first design corresponds to the baseline values, $\mathbf{p}_0$, of the design parameters and the remaining designs are obtained by perturbing the design parameters from their baseline values to their upper limits, one at a time. Let $\mathbf{p}_i, i=1,\cdots,m$ be the vector of parameter values of the i^{th} design. The $(m+1)$ designs are represented by $(m+1)$ points in the space of the design parameters; the first design corresponds to the origin, and the remaining m designs correspond to points located on each of the m axes.

In the PROM approach [7], the mode shapes of a new design, with stiffness and mass matrices $\mathbf{K}_p$ and $\mathbf{M}_p$, are approximated in the subspace $\mathbf{P}$ spanned by the modes of the above $(m+1)$ designs,

$$\mathbf{P} = \left[\hat{\mathbf{\Phi}}_0 \quad \hat{\mathbf{\Phi}}_1 \quad \ldots \quad \hat{\mathbf{\Phi}}_m\right], \tag{4}$$

where $\hat{\mathbf{\Phi}}_0$ is the modal matrix composed of the dominant mode shapes for the design point with all parameters at their lower limits (baseline design), and $\hat{\mathbf{\Phi}}_i$ is the modal matrix for the i^{th}

design point, which is the same with the baseline design except that its i^{th} parameter is at its upper limit. For m design parameters, $(m+1)$ eigenvalue problems must be solved in order to form matrix $\mathbf{P}$. Therefore, both the cost of obtaining modal matrices $\hat{\mathbf{\Phi}}_i$ and the size of matrix $\mathbf{P}$ increase linearly with the number of parameters. Because basis $\mathbf{P}$ captures the dynamic characteristics in each dimension of the parameter space the response at a new design point in the parameter space can be approximated accurately if $\mathbf{P}$ is used in Eq. (3) instead of $\hat{\mathbf{\Phi}}$.

PROM uses the following algorithm to compute the mode shapes of a new design:

1) Select lower and upper bounds of the design parameters.

2) Find the modes of the baseline design and the designs corresponding to the m points in the design space, and form subspace basis $\mathbf{P}$.

3) Condense matrices $\mathbf{K}_p$ and $\mathbf{M}_p$ as

$$\mathbf{K}_R = \mathbf{P}^T \mathbf{K}_p \mathbf{P} \qquad \mathbf{M}_R = \mathbf{P}^T \mathbf{M}_p \mathbf{P}, \tag{5}$$

4) Perform an eigen-analysis on the reduced matrices to calculate eigenvector $\mathbf{\Theta}$.

3) Reconstruct the approximate eigenvectors $\tilde{\mathbf{\Phi}}_p$ using

$$\tilde{\mathbf{\Phi}}_p = \mathbf{P}\mathbf{\Theta}. \tag{6}$$

The computational cost of PROM is very high when the number of parameters m is large because: a) m eigen-analyses must be performed to determine subspace basis $\mathbf{P}$ in Eq. (4), b) $\mathbf{P}$ is usually a very high density matrix, and c) the cost of forming the condensed matrices (by computing the triple matrix products in Eq. (5)) is high.

Balmes et al [7] have proposed a ‘parametric’ approach to reduce the cost, for the special case where matrices $\mathbf{K}_p$ and $\mathbf{M}_p$ can be approximated by the n^{th} order polynomial functions of

m material parameters p_i with matrices $\mathbf{K}_{ij}$ and $\mathbf{M}_{ij}$ as coefficients. For most parameters of material properties used in finite element modeling, a third order polynomial is sufficient. The condensed stiffness and mass matrices can be calculated as follows

$$\mathbf{K}_R = \sum_{i=1}^{m}\sum_{j=0}^{n}\left(\mathbf{P}^T\mathbf{K}_{ij}\mathbf{P}\right)p_i^j \qquad \mathbf{M}_R = \sum_{i=1}^{m}\sum_{j=0}^{n}\left(\mathbf{P}^T\mathbf{M}_{ij}\mathbf{P}\right)p_i^j \ . \qquad (7)$$

Because $\mathbf{K}_{ij}$ and $\mathbf{M}_{ij}$ are constant matrices, the condensations $\mathbf{P}^T\mathbf{K}_{ij}\mathbf{P}$ and $\mathbf{P}^T\mathbf{M}_{ij}\mathbf{P}$ are performed only once and then they are used to form matrices $\mathbf{K}_R$ and $\mathbf{M}_R$ in Eq. (7). The cost of matrix condensation of Eq. (5) is largely reduced because this equation involves only a sum of matrices of reduced size for any design. However, the cost of the parametric approach is still high when the number of parameters m is high because $(m+1)$ eigen-analyses must be performed to form basis **P**.

With the 'parametric' approach, PROM requires first to form the subspace basis $\mathbf{P}$ in Eq. (4), and second to identify matrices $\mathbf{K}_{ij}$ and $\mathbf{M}_{ij}$, and establish the relationship of Eq. (7). The cost of these steps is considered as overhead. The remaining cost of each re-analysis in PROM is minimal. The following two observations can be made. First, the overhead cost is proportional to the number of design parameters, increasing therefore, the efficiency of the PROM method when the number of re-analysis is significantly larger than the number of design parameters. This is the case in simulation-based probabilistic analysis. Second, the efficiency of the PROM approach can be improved by reducing the above overhead cost.

A Kriging method is introduced in [11] to reduce the cost of identifying matrices $\mathbf{K}_{ij}$ and $\mathbf{M}_{ij}$ in Eq. (7), by interpolating the reduced system matrices. In this paper, a modified combined approximations (MCA) method is proposed that can drastically reduce the computational effort

in forming $\mathbf{P}$ for large, finite-element models with a large number of modes. The MCA method is described in Section 3 after a brief introduction of the CA method in the next subsection. We will demonstrate that integration of the MCA and PROM methods realizes considerable computational savings with a small loss of accuracy.

2.3 The Combined Approximations (CA) Method

The PROM method requires an eigenvalue analysis for multiple designs (corner points) to form a basis for approximating the eigenvectors at other designs. This Section provides the fundamentals of the combined approximations (CA) method [15-19] which does not require eigenvalue analyses for multiple designs. Instead, a subspace basis is formed through a recursive process.

If $\mathbf{K}_0$ and $\mathbf{M}_0$ are the stiffness and mass matrices of the original (baseline) design, the exact mode shapes $\mathbf{\Phi}_0$ are obtained by solving the following eigen-problem

$$\mathbf{K}_0\mathbf{\Phi}_0 = \lambda_0\mathbf{M}_0\mathbf{\Phi}_0. \tag{8}$$

We want to approximate the mode shapes of a modified design (subscript p) with stiffness and mass matrices

$$\mathbf{K}_p = \mathbf{K}_0 + \Delta\mathbf{K} \qquad \mathbf{M}_p = \mathbf{M}_0 + \Delta\mathbf{M} \tag{9}$$

where $\Delta\mathbf{K}$ and $\Delta\mathbf{M}$ represent large perturbations. The CA method estimates the new eigenvalues λ_p and eigenvectors $\mathbf{\Phi}_p$ without performing an exact eigenvalue analysis.

The eigen-problem for the modified design can be expressed as

$$\mathbf{\Phi}_p = \lambda_p\mathbf{K}_0^{-1}\mathbf{M}_p\mathbf{\Phi}_p - \mathbf{K}_0^{-1}\Delta\mathbf{K}\mathbf{\Phi}_p, \tag{10}$$

Leading to the following recursive equation

$$\boldsymbol{\Phi}_{p,j} = \lambda_p \mathbf{K}_0^{-1}\mathbf{M}_p\boldsymbol{\Phi}_{p,j-1} - \mathbf{K}_0^{-1}\Delta\mathbf{K}\boldsymbol{\Phi}_{p,j-1} \tag{11}$$

which produces a sequence of approximations of the mode shapes $\boldsymbol{\Phi}_{p,j}, \quad j = 1,2,\ldots,s$. The CA method uses the changes $\mathbf{R}_j = \boldsymbol{\Phi}_{p,j} - \boldsymbol{\Phi}_{p,j-1}$ to form a subspace basis to approximate the modes of the new design. In order to simplify the calculations, $\lambda_p \mathbf{K}_0^{-1}\mathbf{M}_p\boldsymbol{\Phi}_{p,j-1}$ in Eq. (10) is replaced with $\lambda_p \mathbf{K}_0^{-1}\mathbf{M}_p\boldsymbol{\Phi}_0$ and Eq. (11) becomes

$$\boldsymbol{\Phi}_{p,j} = \lambda_p \mathbf{K}_0^{-1}\mathbf{M}_p\boldsymbol{\Phi}_0 - \mathbf{K}_0^{-1}\Delta\mathbf{K}\boldsymbol{\Phi}_{p,j-1}. \tag{12}$$

Eq. (12) shows that the basis vectors satisfy the following recursive equation

$$\mathbf{R}_j = -\mathbf{K}_0^{-1}\Delta\mathbf{K}\mathbf{R}_{j-1} \quad j = 2,\ldots,s, \tag{13}$$

where the first basis vector is assumed to be $\mathbf{R}_1 = \mathbf{K}_0^{-1}\mathbf{M}_p\boldsymbol{\Phi}_0$.

The CA method forms a subspace basis

$$\mathbf{R} = \begin{bmatrix}\mathbf{R}_1 & \mathbf{R}_2 & \cdots & \mathbf{R}_s\end{bmatrix} \tag{14}$$

where s is usually between 3 and 6 [13-17]. The mode shapes of the new ($\mathbf{K}_p$, $\mathbf{M}_p$) design are then approximated in the subspace spanned by $\mathbf{R}_p$ using the following algorithm:

1) Condense the stiffness and mass matrices

$$\mathbf{K}_R = \mathbf{R}^T\mathbf{K}_p\mathbf{R} \qquad \mathbf{M}_R = \mathbf{R}^T\mathbf{M}_p\mathbf{R}, \tag{15}$$

2) Solve the reduced eigen-problem to calculate eigenvector matrix $\boldsymbol{\Theta}$.

3) Reconstruct the approximate eigenvectors of the new design $\tilde{\boldsymbol{\Phi}}_p$ as

$$\tilde{\boldsymbol{\Phi}}_p = \mathbf{R}\boldsymbol{\Theta}. \tag{16}$$

The eigenvalues of the new design are approximated by the eigenvalues $\tilde{\lambda}_p$ of the reduced eigen-problem.

The CA method has three main advantages; a) it only requires a single matrix decomposition of stiffness matrix $\mathbf{K}_0$ in Eq. (13) to calculate the subspace basis $\mathbf{R}$, b) it is accurate because the basis is updated for every new design, and c) the eigenvectors of a new design are efficiently approximated by Eq. (16) where the eigenvectors $\boldsymbol{\Theta}$ correspond to a much smaller reduced eigen-problem. However if a large number of re-analyses are needed, the computational cost can be high because a new basis must be calculated in CA for every re-analysis. Note that steps (15) and (16) are similar to steps (5) and (6) of PROM. The former use basis $\mathbf{R}$ and the latter use basis $\mathbf{P}$.

The CA method is more efficient than PROM for design problems where few re-analyses are required for two reasons. First, it does not require calculation of the eigenvectors $\boldsymbol{\Phi}_i$, $i = 1, \cdots, m$, of the m corner design points, and second the cost of matrix condensation of Eq. (15) is much lower than that of Eq. (5), because the size (number of columns) of basis $\mathbf{R}$ is not proportional to the number of parameters m as in basis $\mathbf{P}$.

As discussed in Section 2.2, the PROM approach is efficient only when the 'parametric' relationship is already established because a large overhead cost, proportional to the number of design parameters, is required. On the contrary, the CA method does not require such an overhead cost because the re-analysis cost is not proportional to the number of design parameters. The CA method is therefore, more suitable than PROM, if the number of re-analyses is less than or comparable to the number of design parameters.

The CA method can however, become expensive when many re-analyses are needed because a new basis $\mathbf{R}$ and the condensed mass and stiffness matrices in Eq. (15) must be calculated for each re-analysis. Examples where many analyses are needed are optimization problems in which a Genetic Algorithm is employed to search for the optimum, and probabilistic analysis problems in which Monte-Carlo simulation is used. The PROM method can be more suitable for these problems because the subspace basis $\mathbf{R}$ does not change for every new design point.

In the literature, the accuracy and efficiency of the CA method has been mostly tested on problems involving structures with up to few thousands of DOFs, such as frames or trusses [12-19]. Moreover to the best of the authors' knowledge, it has not been integrated with commercial finite element codes. We have tested the CA method using the structural dynamics response of a medium size (65,000 DOF), finite-element model. Due to its high modal density, there were more than two hundred dominant modes in the frequency range of zero to 50 Hz. It was observed that the computational savings of the CA method, using the recursive process of Eq. (13), were not substantial. For this reason, we introduce in this paper a modified combined approximations method (MCA) by modifying the recursive process of Eq. (13). The new method, which is described in Section 3.1, is much more efficient than the original CA method for large size models.

3. A NEW METHOD FOR EFFICIENT RE-ANALYSIS

3.1 Modified Combined Approximation (MCA) Method

The cost of calculating the subspace basis in Eq. (13) consists of one matrix decomposition (DCMP) and one forward-backward substitution (FBS). The DCMP cost is only related to the

size and density of the symmetric stiffness matrix, while the FBS cost depends on both the size and density of the stiffness matrix and the number of columns of $\mathbf{\Phi}_0$.

As we mentioned in Section 2.1, the higher the frequency range of interest the more modes are needed to predict the structural response accurately. In such a case, although a single DCMP is needed in Eq. (13), the number of columns in $\mathbf{\Phi}_0$ may increase drastically, thereby increasing the cost of the repeated FBS. When the number of dominant modes becomes very large, the cost of performing the calculations in Eq. (13) can be dominated by the FBS cost. For example, the vehicle model of Section 4.2 (Figure 5) has about 65,000 degrees of freedom, and 1050 modes in the frequency range of 0-300 Hz. If $\mathbf{\Phi}_0$ has only one mode, the cost of one DCMP is 1.1 seconds and the cost of one FBS is less than 0.1 seconds. In this case, the total cost is dominated by the DCMP, and the CA method reduces the cost of one re-analysis considerably. However, if $\mathbf{\Phi}_0$ has 1050 modes the cost of FBS increases to 29 seconds, dominating the cost of the DCMP.

The CA method can therefore, improve the efficiency only when the number of retained modes is small. Otherwise, the computational savings do not compensate for the loss of accuracy from using $\mathbf{K}_0$ (stiffness matrix of baseline design) instead of $\mathbf{K}_p$ (stiffness matrix of new design). For this reason, a modified combined approximations (MCA) method is proposed in this paper.

The MCA method uses a subspace basis $\mathbf{T}$ whose columns are constructed using the recursive process

$$\begin{aligned} \mathbf{T}_1 &= \mathbf{K}_p^{-1}\left(\mathbf{M}_p\mathbf{\Phi}_0\right) \\ \mathbf{T}_i &= \mathbf{K}_p^{-1}(\mathbf{M}_p\mathbf{T}_{i-1}) \qquad i = 2,3,\cdots,s \end{aligned} \tag{17}$$

instead of that of Eq. (13). The selection of the appropriate number of basis vectors s is

discussed later. The only difference between Eqs. (13) and (17) is that matrix $\mathbf{K}_0$ is inverted in the former while matrix $\mathbf{K}_p$ is inverted in the latter. The DCMP of $\mathbf{K}_p$ must be repeated for every new design. However, the cost of the repeated DCMP does not significantly increase the overall cost in Eq. (17), because the latter is dominated by the FBS cost. Note that the iterative process of Eq. (17) provides a continuous mode shape updating of the new design. If the process converges in *s* iterations, the mode shapes $\mathbf{T}_s$ will be the exact mode shapes $\mathbf{\Phi}_p$. However, Eq. (13) does not have the same property. For this reason, the vectors $\mathbf{T}_i$ provide a more accurate approximation of the exact mode shapes $\mathbf{\Phi}_p$ than the $\mathbf{R}_i$ vectors of the original CA method. This is an important advantage of MCA.

Because the mode shapes $\mathbf{T}_i$ in Eq. (17) can quickly converge to the exact mode shapes $\mathbf{\Phi}_p$, only one iteration (i.e. *s*=1) may be needed, resulting in

$$\mathbf{T} = \mathbf{T}_1 \ . \tag{18}$$

If the convergence is slow, multiple sets of updated mode shapes can be used so that

$$\mathbf{T} = \begin{bmatrix} \mathbf{\Phi}_0 & \mathbf{T}_1 & \mathbf{T}_2 & \cdots & \mathbf{T}_s \end{bmatrix}. \tag{19}$$

For better accuracy, the above basis can also include the mode shapes $\mathbf{\Phi}_0$ of the baseline design. The approximate modes $\mathbf{T}_i$ are more accurate than the CA vectors $\mathbf{R}_i$ in approximating the exact mode shapes $\mathbf{\Phi}_p$, because modes $\mathbf{T}_i$ are updated for each new design. MCA can therefore, achieve similar accuracy to the CA method using fewer modes. As we will demonstrate later using a vehicle model, the MCA method achieves good accuracy with only 1 or 2 basis vectors, whereas the CA method requires 3 to 6 basis vectors [13-17].

In summary, the proposed MCA method involves four steps in calculating the approximate

eigenvectors $\tilde{\mathbf{\Phi}}_p$:

1) Find basis **T** using Eq. (18) or Eq. (19).

2) Calculate the condensed stiffness and mass matrices $\mathbf{K}_p$ and $\mathbf{M}_p$

$$\mathbf{K}_R = \mathbf{T}^T \mathbf{K}_p \mathbf{T} \qquad \mathbf{M}_R = \mathbf{T}^T \mathbf{M}_p \mathbf{T}. \tag{20}$$

3) Solve the following reduced eigen-problem to calculate the eigenvalues and the projections of the modes in the reduced space spanned by **T**

$$(\mathbf{K}_R - \lambda \mathbf{M}_R)\mathbf{\Theta} = 0. \tag{21}$$

4) Reconstruct the approximate eigenvectors $\tilde{\mathbf{\Phi}}_p$ as

$$\tilde{\mathbf{\Phi}}_p = \mathbf{T}\mathbf{\Theta}. \tag{22}$$

it should be noted that the slightly increased cost of using Eq. (19) instead of Eq. (18) is usually smaller than the realized savings in steps (20) through (22) due to the smaller size of the reduced basis **T**. The bases of Eqs. (18) and (19) are smaller in size than the CA basis of Eq. (14) for comparable accuracy. The MCA method requires therefore, less computational effort for steps (20) through (22). The computational savings compensate for the increased cost of DCMP for dynamic re-analysis of large finite-element models with a large number of dominant modes.

All mode shapes in Eq. (21) must be calculated simultaneously in order to ensure that the approximate mode shapes $\tilde{\mathbf{\Phi}}_p$ are orthogonal with respect to the mass and stiffness matrices. However, the cost of estimating the mode shapes $\tilde{\mathbf{\Phi}}_p$ in Eqs. (20) to (22) may increase quickly (quadratically) with the number of modes, and as a result, the MCA method may become more expensive than a direct eigen-solution when the number of dominant modes exceeds a certain limit.

One way to circumvent this problem is to divide the frequency response into smaller frequency bands and calculate the frequency response in each band separately instead of solving for the frequency response in one step. The modal basis $\mathbf{T}$ in Eq. (19) is divided into k groups as

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}^1 & \mathbf{T}^2 & \cdots & \mathbf{T}^k \end{bmatrix} \tag{23}$$

where,

$$\mathbf{T}^i = \begin{bmatrix} \mathbf{\Phi}_0^i & \mathbf{T}_1^i & \mathbf{T}_2^i & \cdots & \mathbf{T}_s^i \end{bmatrix}. \tag{24}$$

Each group $\mathbf{T}^i$ contains roughly n/k original modes $\mathbf{\Phi}_0^i$ from $\mathbf{\Phi}_0$, and their corresponding improved modes. The eigenvectors of the new design are calculated using $\mathbf{T}^i$ instead of $\mathbf{T}$ in Eqs. (20) to (22). The process is repeated k times using a modal basis that is $1/k$ of the size of the original modal basis. All k groups of eigenvectors are then collected together to calculate the frequency response of the new design. As demonstrated in the numerical examples section, this reduces the cost considerably with minimal loss of accuracy.

3.2 Integration of the MCA Method with Optimization

We have mentioned that the MCA method provides a good balance between accuracy and efficiency for problems that require a moderate number of re-analyses, as in gradient-based optimization problems. For problems where a large number of re-analyses is necessary, such as in probabilistic analysis and gradient-free (e.g. genetic algorithms) optimization, a combination of the MCA and PROM methods is more suitable than the MCA method.

In this paper, we have integrated the MCA method with the gradient-based optimizer in NASTRAN v2001. Figure 1 shows a flowchart of the optimization process. The DMAP (Direct Matrix Abstraction Program) capabilities in NASTRAN have been used to integrate the MCA

method and the NASTRAN modal dynamic response and optimization (Sol. 200). The highlighted boxes indicate modifications to the NASTRAN optimizer. Section 4.2 demonstrates how this process was used to optimize the vibro-acoustic behavior of a 65,000 DOF, finite-element model of a truck. Using the MCA method, the computational cost of the entire optimization process was reduced in half compared with the existing NASTRAN approach.

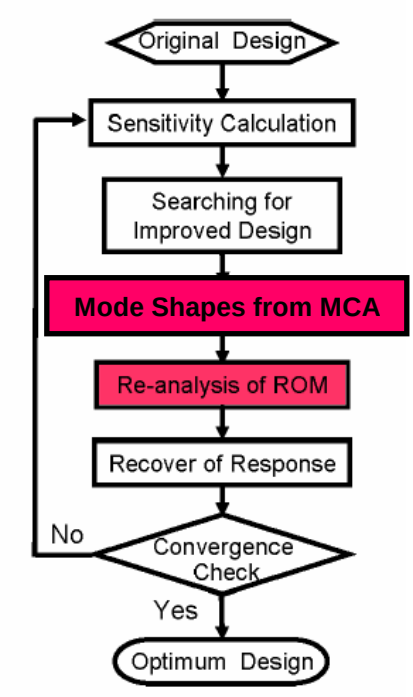

Figure 1. Flowchart for MCA-enhanced NASTRAN optimization

3.3 Integration of MCA and PROM Methods

The PROM method requires exact calculation of the mode shapes for all designs corresponding to the corner points of the parameter space in order to calculate the subspace basis $\mathbf{\Phi}$. The required computational effort can be prohibitive if a large number of parameters (optimization design variables) is used. This effort can be reduced substantially if the modes of each corner point are approximated by the MCA method. In this case, an exact eigen-solution is

required only for the baseline design.

The following steps describe an algorithm to integrate the MCA and PROM methods:

1. Perform an exact eigen-analysis at the baseline design point $\mathbf{p}_0$ to obtain the baseline mode shapes, $\mathbf{\Phi}_0$.

2. Use the MCA method at design point $\mathbf{p}_i$, where all parameters are set at their low limits except the i^{th} parameter which is set at its upper limit. For point $\mathbf{p}_0$, all parameters are at their lower limits. Obtain approximate mode shapes for the i^{th} corner point using the following recursive process

$$\begin{aligned} \mathbf{T}_{i,1} &= \mathbf{K}_i^{-1}\left(\mathbf{M}_i\mathbf{\Phi}_o\right) \\ \mathbf{T}_{i,j+1} &= \mathbf{K}_i^{-1}(\mathbf{M}_i\mathbf{T}_{i,j}) \qquad j = 2,3,\cdots,s \end{aligned} . \tag{25}$$

3. Form the subspace basis T as

$$\mathbf{T} = \left[\mathbf{\Phi}_o \quad \mathbf{T}_{0,s} \quad \mathbf{T}_{1,s} \quad \cdots \quad \mathbf{T}_{m,s}\right] \tag{26}$$

where m is the total number of parameters.

4. Obtain the approximate mode shapes $\tilde{\mathbf{\Phi}}_p$ using the subspace projection procedure of Eqs. (5) and (6), where $\mathbf{T}$ is used instead of $\mathbf{P}$. $\tilde{\mathbf{\Phi}}_p$ can be subsequently used in a modal dynamic response solution. Only step 4 is repeated in re-analysis. The computational savings can be substantial especially for problems where many re-analyses are needed, such as in probabilistic analysis of structural dynamic systems using Monte-Carlo simulation.

4. NUMERICAL EXAMPLES

A simple frame model is first used to compare the accuracy and efficiency of the MCA

method with that of the original CA method. Subsequently, a vehicle model is used to demonstrate the advantages of the combined MCA and PROM method in optimizing the vibro-acoustic response of a vehicle. Both examples are performed on a SUN ULTRA workstation using NASTRAN v2001. The CA, MCA and PROM methods are implemented in NASTRAN DMAP.

4.1 A Simple Frame Model

The MCA method is applied on the simple eight-story frame structure of Figure 2. The same frame model has been extensively used in the literature on the CA method [16-18]. A lumped mass representation is used with each story girder having a mass *M*. The girders are assumed to be non-deformable with the initial lateral stiffness $\left(EI / L^3\right)$ of 50.0 (kips/inch) for stories 1 and 2, 55.0 for stories 3 to 6 and 60.0 for stories 7 and 8. The corresponding girder lengths are 2.3208 ft for stories 1 and 2, 2.2482 ft for stories 3 to 6 and 2.1840 ft for stories 7 and 8.

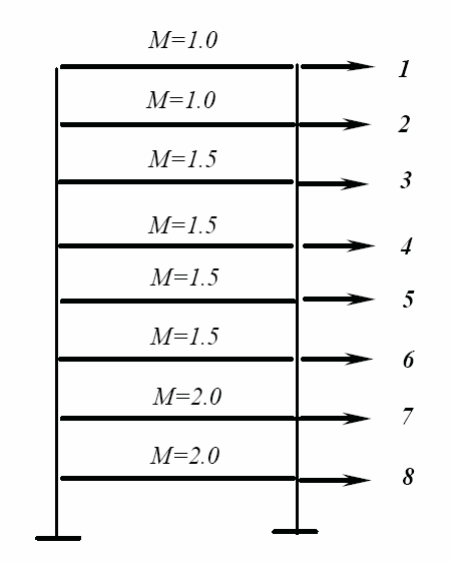

Figure 2. Eight-story frame model

In [16-18], the Young's moduli of all girders are treated as design parameters, assuming implicitly that among different designs, the stiffness matrix varies linearly with the Young's moduli and the mass matrix remains unchanged. In this example, we allow the girder lengths to vary resulting in a nonlinear change of the stiffness matrix and a linear change of the mass matrix from the baseline. This increases the complexity of the re-analysis.

For the modified design, each girder mass is increased by 25 percent, and each girder stiffness is increased by a factor of 1.953. Table 1 shows the first mode shapes $\boldsymbol{\varphi}_p$ and $\boldsymbol{\varphi}_o$ for the modified and baseline designs respectively obtained from NASTRAN. These results are considered exact in this study.

Table 1. Accuracy of mode shapes for the frame model

Story #	$\boldsymbol{\varphi}_p$	$\boldsymbol{\varphi}_o$		$\mathbf{t}_1$		$\mathbf{t}_2$		$\tilde{\boldsymbol{\varphi}}_p$	
		Disp.	**Err(%)**	**Disp.**	**Err(%)**	**Disp.**	**Err(%)**	**Disp.**	**Err(%)**
1	1.0000	1.0000	0	1.0000	0	1.0000	0	1.0000	0
2	0.9607	0.9700	0.97	0.9592	0.16	0.9603	0.04	0.9583	0.25
3	0.8836	0.9109	3.09	0.8787	0.55	0.8825	0.12	0.8761	0.85
4	0.8235	0.8199	0.43	0.8153	0.99	0.8220	0.18	0.8149	1.03
5	0.7407	0.6954	6.12	0.7286	1.64	0.7388	0.26	0.7313	1.27
6	0.6376	0.5424	14.93	0.6219	2.46	0.6355	0.34	0.6285	1.43
7	0.5170	0.3673	28.97	0.4999	3.32	0.5149	0.42	0.5109	1.20
8	0.2673	0.1883	29.53	0.2563	4.09	0.2659	0.51	0.2620	1.98

The MCA method was used to approximate the mode shapes of the modified design, using the recursive process of Eq. (23) with one or two iteration steps ($s = 1 \text{ or } 2$). In Table 1, $\mathbf{t}_1$ and

$\mathbf{t}_2$ are the once ($s=1$) and twice ($s=2$) updated mode shapes. Mode shape $\mathbf{t}_2$ is very close to the exact mode shape $\boldsymbol{\varphi}_p$. The maximum error between the two is only 0.5%. A subspace basis $\mathbf{T}=[\boldsymbol{\varphi}_o \quad \mathbf{t}_1]$ (similar to Eq. 19) was also tested. The resulting approximate mode shape is indicated by $\tilde{\boldsymbol{\varphi}}_p$ in Table 1. While the accuracy of $\tilde{\boldsymbol{\varphi}}_p$ is not as good as $\mathbf{t}_2$, it is more accurate than $\mathbf{t}_1$, indicating that the basis of Eq. (19) provides better accuracy than that of Eq. (18) as expected.

Figure 3, compares $\mathbf{t}_1$ (MCA with once-updated mode shapes), CA with twice-updated mode shapes, and $\boldsymbol{\varphi}_p$ (exact analysis using NASTRAN). The baseline mode is also shown. It is observed that $\mathbf{t}_1$ is closer to $\boldsymbol{\varphi}_p$ than the twice-updated mode from the CA method. This supports the claim of Section 3.1 that the MCA basis $\mathbf{T}_i$ provides more accurate results than the CA basis $\mathbf{R}_i$.

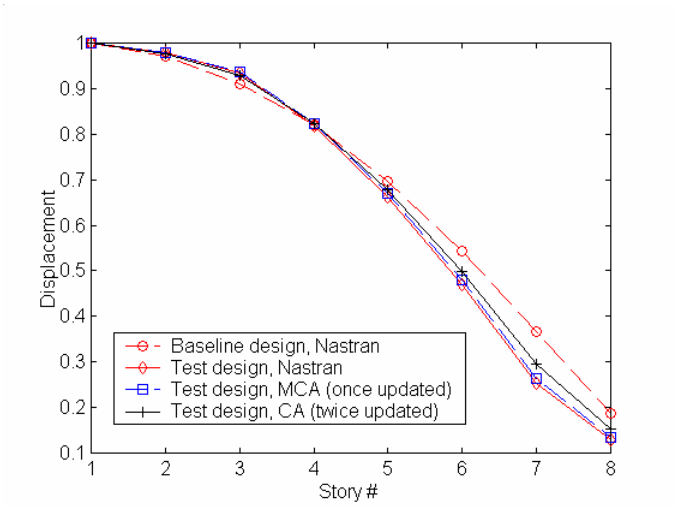

Figure 3. Mode shape comparison among NASTRAN, MCA and CA methods

4.2 A Vehicle Model

Figure 4 shows a 65,000 DOF finite-element model of a pickup truck. The sound pressure level at the driver's ear location is calculated using a vibro-acoustic analysis. The structural forced vibration response due to unit harmonic forces in x, y, and z directions at the engine mount locations, is coupled with an interior acoustic analysis. The first and second eigen-frequecies of the acoustic volume inside the cabin are 95.9 Hz and 128.3 Hz. The sound pressure level is calculated in the 80 to 140 Hz frequency range.

For this vibro-acoustic analysis, the structure and fluid domains are coupled through boundary conditions ensuring continuity of vibratory displacement and acoustic pressure. A finite-element formulation of the coupled undamped problem yields the following system equations of motion [21]

$$\left(\begin{bmatrix} \mathbf{K}_S & -\mathbf{H}_{SF} \\ 0 & \mathbf{K}_F \end{bmatrix} - j\omega^2 \begin{bmatrix} \mathbf{M}_S & 0 \\ \rho_0 c_0^2 \mathbf{H}_{SF}^T & \mathbf{M}_F \end{bmatrix} \right) \begin{bmatrix} \mathbf{d}_S \\ \mathbf{p}_F \end{bmatrix} = \begin{bmatrix} \mathbf{f}_b \\ \mathbf{f}_q \end{bmatrix}, \tag{27}$$

where the primary variables are the vibratory displacement $\mathbf{d}_S$ and the acoustic pressure $\mathbf{p}_F$. The finite-element representation of the two domains consists of stiffness and mass matrix pairs $\langle \mathbf{K}_S, \mathbf{M}_S \rangle$ and $\langle \mathbf{K}_F, \mathbf{M}_F \rangle$, respectively. The air density and wave speed are ρ_0 and c_0. External forces are shown in the right hand side of Eq. (27).

The spatial coupling matrix $\mathbf{H}_{SF}$ indicates coupling between the two domains. Due to this coupling, which is usually referred to as "two-way coupling", the combined structural-acoustic system of equations is not symmetric. If the acoustic effect on the structural response is small, the coupling term can be omitted, resulting in the so-called "one-way coupling," where the structural response is first calculated and then used as input ($\mathbf{f}_q$ in Eq. 27) to solve for the acoustic

response.

The coupled structure-acoustic system can be solved either by a direct method, or more efficiently by a modal response method as described in Section 2.1. A modal response method can be applied to both the structural and the acoustic domains.

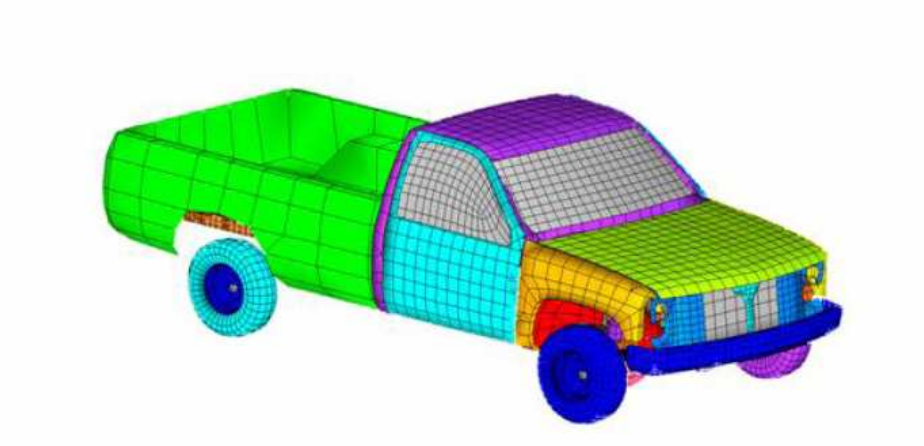

Figure 4. A pickup truck model

4.2.1. Combined MCA and PROM Methods

A re-analysis was performed for a modified design for which the following five plate thickness parameters vary; chassis and its cross links, cabin, truck bed, left door, and right door. All parameters were increased by 100% of their baseline values.

The sound pressure at the driver's ear was calculated using "two-way" coupling. A structural modal frequency response analysis was used. The acoustic response was calculated using a direct method because the size of the acoustic model is relatively small. For the structural analysis, 1050 modes were retained in the 0 to 300 Hz frequency range. The combined MCA and PROM approach was compared against the NASTRAN direct solution for a modified design where all

five parameters were at their upper limits. Only one iteration was performed in Eq. (17) in order to get one set of once-updated mode shapes for each corner design point. The subspace basis, which includes information for all five design parameters, is therefore, represented by

$$\mathbf{T} = \left[\boldsymbol{\Phi}_o \quad \mathbf{T}_{0,1} \quad \mathbf{T}_{1,1} \quad \cdots \quad \mathbf{T}_{5,1}\right]. \tag{28}$$

Figure 5 shows the percent error in natural frequencies as predicted by the combined MCA and PROM method and NASTRAN. The maximum error is less than 0.45% in the entire frequency range. Figure 6 indicates that the sound pressures calculated by both methods are almost identical. The computational effort for the MCA method to obtain approximate mode shapes at each corner design point is about 30 seconds. In contrast, it takes about 180 seconds for an exact eigen-solution using NASTRAN.

The computational costs of constructing the reduced basis ($\mathbf{P}$ in PROM method and $\mathbf{T}$ in PROM+CA method) are compared in Table 2. For the case of 5 design parameters, the total cost was reduced from 1080 seconds to 330 seconds. The computational saving is more significant if the number of design parameter increases.

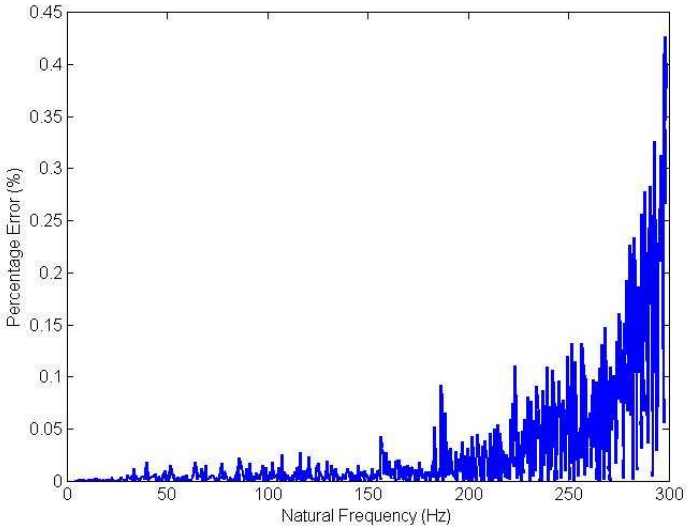

Figure 5. Percent error of natural frequencies between combined MCA and PROM method and NASTRAN

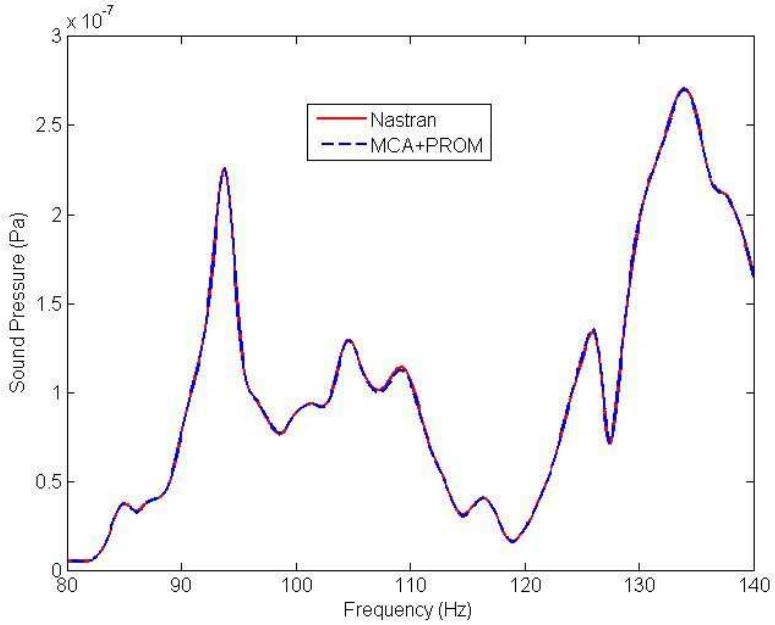

Figure 6. Comparison of sound pressure at driver's ear between combined MCA and PROM method and NASTRAN

Table 2. CPU time for constructing reduced basis

Method	**Solving for mode shape Φ_0 at baseline design**	**Solving for mode shapes at 5 corner design points**	**Total Cost**
PROM	180 sec.	180*5=900 sec.	1080 sec.
PROM+MCA	30 sec	30*5=150 sec	330 sec.

4.2.2 Optimization using MCA Method

The objective here is to minimize the sound pressure at the driver's ear. In order to

demonstrate the capability of the MCA method to handle a large number of parameters, a total of 41 design parameters are used representing the thickness of all vehicle components modeled with plate elements. All thicknesses are allowed to change by 100% from their baseline values. Table 3 gives a description of all design parameters. All design parameters are at their low bound for the optimization initial point.

Table 3: Description of design parameters

Prm. #	**Description (thickness of)**	**Prm. #**	**Description (thickness of)**	**Prm. #**	**Description (thickness of)**
1	Bumper	15	Radiator mtg.	29	Tire, front right
2	Rails	16	Radiator mtg., mid.	30	Tire, rear left
3	A-arm, low left	17	Fan cover, low	31	Tire, rear right
4	A-arm, low right	18	Fan cover, up	32	Engine outer
5	A-arm, up left	19	Cabin	33	A-arm conn., up left
6	A-arm, up right	20	Cabin mtg. reinf.	34	A-arm conn., up right
7	Tire rim	21	Door, left	35	A-arm conn., low left
8	Engine Oil-box	22	Door, right	36	A-arm conn., low right
9	Fan	23	Bed	37	Glass, left
10	Hood	24	Brake, front left	38	Glass, right
11	Fender, left	25	Brake, front right	39	Glass, rear
12	Fender, right	26	Rail conn., rear	40	Glass, front
13	Wheel house, left	27	Rail mount	41	Rail conn., front
14	Wheel house, right	28	Tire, front left		

The MCA method approximates the mode shapes at intermediate design points, using only $\mathbf{T}_1$ in Eq. (17). Therefore, the subspace basis at each optimization step is

$$\mathbf{T} = \left[\boldsymbol{\Phi}_o \quad \mathbf{T}_1\right]. \tag{29}$$

Because 1050 modes exist in the frequency range of 0 to 300 Hz of the initial design, the size of the MCA modal basis is 2100.

The cost of solving for 1050 modes directly from NASTRAN is 180 seconds. In the MCA method, cost of solving the linear system of equations in Eq. (17) is 31 seconds, and the additional combined cost of Eqs. (20) to (22) is 373 seconds, resulting in a total cost of 404 seconds. For this reason, the 1050 modes are divided into 21 groups and the modes in each group are obtained separately as explained in the latter part of Section 3.1. This reduces the cost of Eqs. (20) to (22) to 66 seconds. The total cost of the MCA method is then 97 seconds, which is about half of the cost of the direct NASTRAN method. Details of the computational costs are provided in Table 3.

Table 3. CPU time of the MCA method

	k=1	**k=21**
Eq. (17)	31 sec.	31 sec.
Eq. (20)	258 sec.	50 sec.
Eq. (21)	48 sec.	6 sec.
Eq. (22)	67 sec.	10 sec.
Total Cost	404 sec.	97 sec.

The optimization process of Figure 1 is implemented in NASTRAN, using DMAP. The gradient-based optimizer in NASTRAN (Sol 200) needed three iterations to calculate the optimal design. Figure 7 compares the sound pressure at the driver's ear between the optimal design and the initial design. Figure 8 indicates the percentage increase of optimal values relative to the initial values for all 41 design parameters.

The Sequential Quadratic Programming (SQP) algorithm of NASTRAN can find only a

local optimum instead of the global optimum. In the frequency range of 80-140Hz, the maximum sound pressure is slightly reduced from 7.9E-7 to 7.2E-7 Pascal. Most parameters are only slightly changed. The largest increase is 20% for the rail mount thickness (parameter #27).

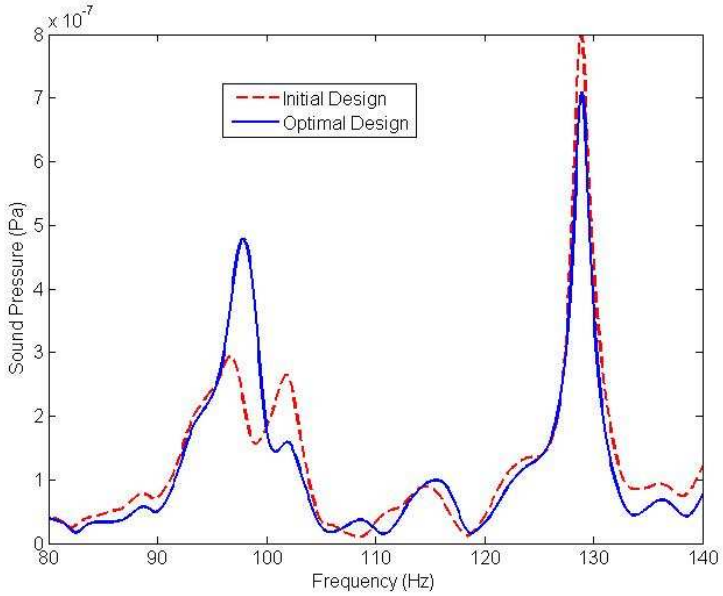

Figure 7. Comparison of sound pressure at driver's ear, between initial and optimal designs

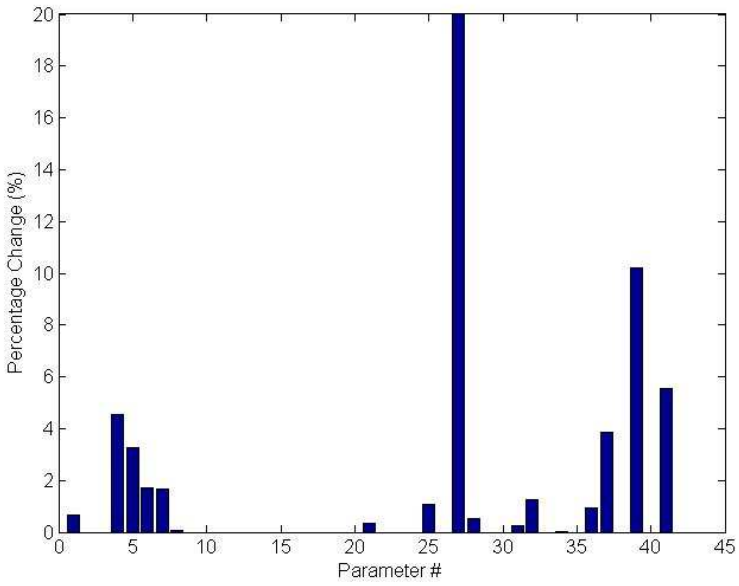

Figure 8. Percent increase of optimal design parameters relative to baseline design parameters

To obtain a more significant design improvement, two additional studies are performed using the MCA method in a Genetic Algorithm. In the first study, 20 initial populations and 4 generations were used, and in the second study 100 initial populations and 6 generations were used. Figures 9 and 10 show that the optimization results are affected by the number of initial populations and the number of generations. While more number of initial populations and generation produces a slightly better result, both studies have achieved much better results than the SQP algorithm. In the case of 100 initial populations and 6 generations, the sound pressure is reduced from 7.9E-7 Pascal to 2.0E-7 Pascal, which is equivalent to about 15 dB in sound pressure level (SPL).

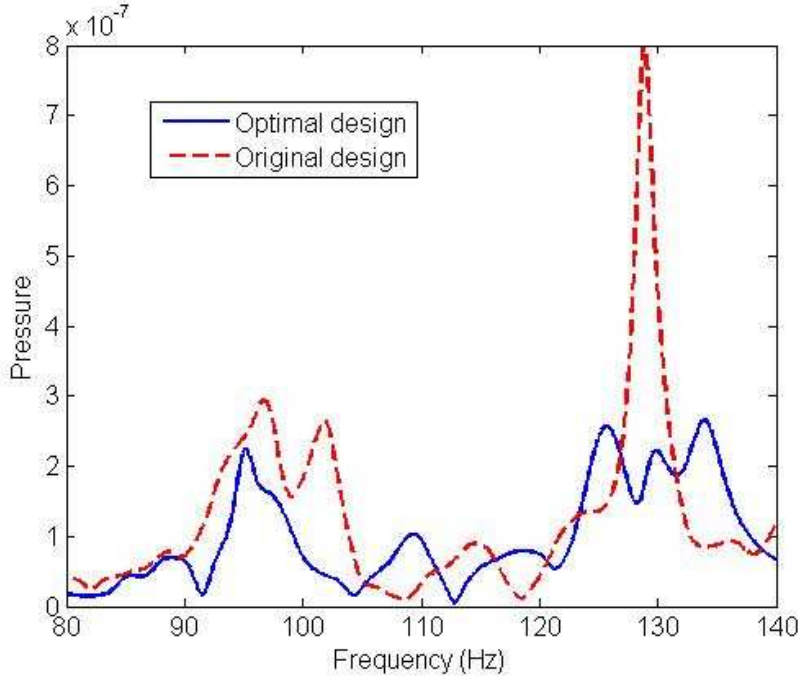

Figure 9. Comparison of sound pressure at driver's ear, between baseline and optimal designs, with 20 initial populations and 4 generations

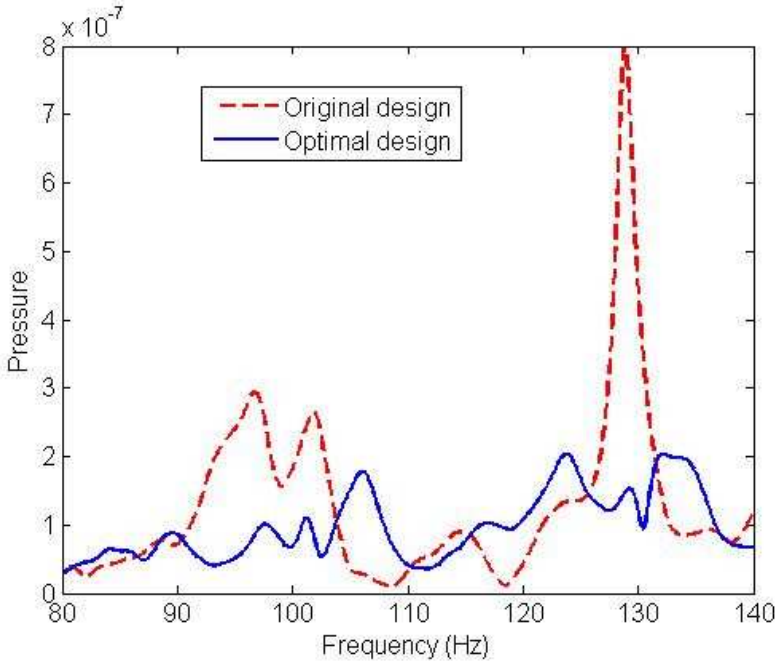

Figure 10. Comparison of sound pressure at driver's ear, between baseline and optimal designs, with 100 initial populations and 6 generations

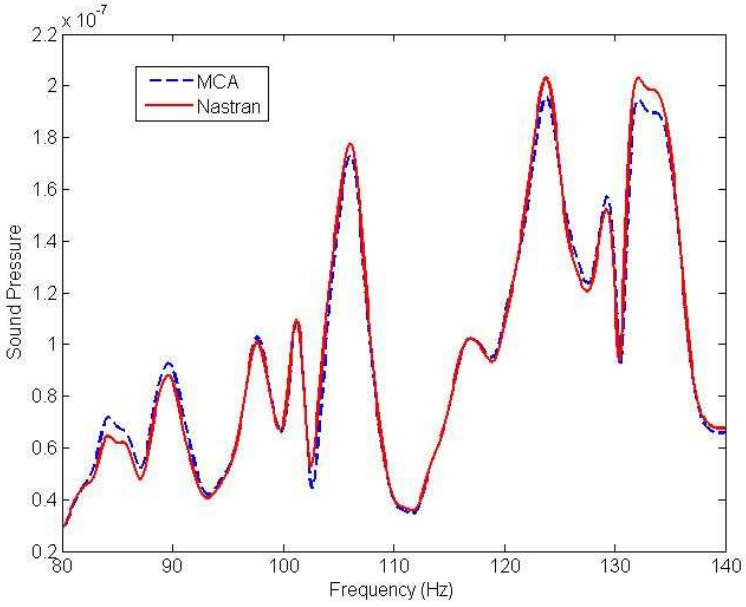

Figure 11. Comparison of sound pressure at driver's ear, between direct NASTRAN and MCA

To verify the accuracy of the MCA approximation, the sound pressure response of the optimal design from MCA+GA with 100 initial populations and 6 generations was evaluated by both direct NASTRAN and MCA. According to Figure 11, the MCA method is very accurate.

For a comparable to MCA accuracy, the original CA method needed three sets of mode shapes to form the subspace basis, requiring 90 seconds to solve the linear equations. The much larger mode basis **R** in CA increases the computational cost in calculating the triple matrix products of Eq. (15). Therefore for large scale, finite-element models with a high modal density, the proposed MCA method can be more efficient compared to a complete NASTRAN analysis and the original CA method.

5. SUMMARY AND CONCLUSIONS

An accurate and efficient modified combined approximations (MCA) re-analysis method has been proposed for approximate eigenvalue analysis of large size models. The method was used to improve the computational efficiency of the parametric reduced-order modeling (PROM) method. The proposed method has been implemented in NASTRAN and demonstrated on analysis and design optimization of two structures; a simple frame representing a building and a real-world large-scale model of a truck. We showed that the results from the proposed MCA method are very similar to those from a complete NASTRAN analysis but at less than 50% of the NASTRAN cost.

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